Section 12.4: Parallel and Perpendicular Vectors: Dot Product

Definition of the Dot Product:

\[ a \cdot b = (a_1, a_2) \cdot (b_1, b_2) = a_1b_1 + a_2b_2 \]

→ also known as the scalar product or inner product

→ \( a \cdot b \) is a one "number" answer
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Properties of the Dot Product: If \( \mathbf{a} \), \( \mathbf{b} \), and \( \mathbf{c} \) are vectors and \( m \) is a real number then:

1) \( \mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2 \)
2) \( \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} \)
3) \( \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} \)
4) \( m(\mathbf{a} \cdot \mathbf{b}) = (m\mathbf{a}) \cdot \mathbf{b} = \mathbf{a} \cdot (m\mathbf{b}) \)
The Angle Between Two Vectors:
If $\theta$ is the angle between two nonzero vectors $a$ and $b$, then:

$$\cos \theta = \frac{a \cdot b}{|a| \cdot |b|}$$

in other words...

$$\cos \theta = \frac{a_1 b_1 + a_2 b_2}{\sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2}}$$
Orthogonal Vectors:
Two vectors are orthogonal (perpendicular) if and only if $\mathbf{a} \cdot \mathbf{b} = 0$

In other words...

two vectors are perpendicular if their DOT PRODUCT is ZERO

Parallel Vectors:
Two vectors are parallel if they are "multiples" of each other

$\overrightarrow{a} = (2, 4)$
$\overrightarrow{b} = (4, 8)$
$\overrightarrow{c} = (1, 2)$
$\overrightarrow{d} = (-2, -4)$

\[ \overrightarrow{e} = (4, 16) \]

Not Parallel

$4 \times 2 \quad 16 \times 4$
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Example: Let \( \mathbf{a} = (8, -4) \) and \( \mathbf{b} = (2, 1) \).

a) Show that \( \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} \)

\[
\mathbf{a} \cdot \mathbf{b} = 8 \cdot 2 + (-4) \cdot 1 = 16 - 4 = 12
\]
\[
\mathbf{b} \cdot \mathbf{a} = 2 \cdot 8 + 1 \cdot (-4) = 16 - 4 = 12
\]

\[12 = 12\]

b) Find the angle \( \theta \) between \( \mathbf{a} \) and \( \mathbf{b} \) to the nearest tenth of a degree.

\[
\mathbf{a} = (8, -4) \quad \mathbf{b} = (2, 1)
\]

\[
\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{12}{\sqrt{8^2 + (-4)^2} \cdot \sqrt{2^2 + 1^2}}
\]

\[
\cos \theta = \frac{12}{\sqrt{80} \cdot \sqrt{5}}
\]

\[
\cos \theta = \frac{12}{\sqrt{400 \cdot 5}}
\]

\[
\cos \theta = 0.60
\]

\[
\theta = \cos^{-1}(\text{ANS}) \approx 53.1^\circ
\]

c) Find a vector that is parallel to \( \mathbf{a} \).

\[
\overrightarrow{\text{a}} = (8, -4)
\]

\[
(16, -8) \cdot (24, -12) \cdot (4, -2)
\]

d) Find a vector that is perpendicular to \( \mathbf{a} \).

\[
\overrightarrow{\mathbf{a}} = (8, -4)
\]

\[
\overrightarrow{\mathbf{r}} = (x, y)
\]

\[
\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{r}} = 8x + (-4)y = 0
\]

\[
8x - 4y = 0
\]

\[
8(1) - 4(2) = 0 \quad \overrightarrow{\mathbf{r}} = (1, 2)
\]

\[
8(2) - 4(4) = 0 \quad \overrightarrow{\mathbf{r}} = (2, 4)
\]

\[
8(4) - 4(-2) = 0 \quad \overrightarrow{\mathbf{r}} = (-1, -2)
\]
Assign #3

Pg. 444:
1-21 odd