Section 4.8: Problem Solving using Polynomial Equations

Remember...

Four Step Process for Solving Word Problems

Step 1: Define the Variable

Step 2: Write an Equation

Step 3: Solve the Equation

Step 4: Write a Sentence
Section 4.8: Problem Solving using Polynomial Equations

Example #1: The sum of two numbers is 9. The sum of their squares is 101. Find the numbers.

Step 1: Let $x = 1^{st}$ number and $y = 2^{nd}$ number.

Step 2: $x + y = 9$ \[x = 9 - y\]

$x^2 + y^2 = 101$

Step 3: $(9-y)^2 + y^2 = 101$

$(9-y)(9-y)+y^2=101$

$81 -9y -9y + y^2 + y^2 = 101$

Step 4: $2y^2 - 18y - 20 = 0$

$2(y^2 - 9y - 10) = 0$

$2(y-10)(y+1) = 0$

$y^2 + 10y - 10$

$y = 10, y = -1$

$x = 9 - y$

$x = 9 - 10$

$x = -1$ or $x = 10$

Step 4: The numbers could be $-1$ and $10$ or $10$ and $-1$. 
Example #2: The length of a rectangle is 1 meter less than twice the width. If the area is $55 \text{ m}^2$, find the perimeter.

**Step 1:**

\[ A = l \times w \]
\[ P = \text{add all sides} \]
\[ x = \frac{w}{2} = 5\frac{1}{2} \]
\[ 2x - 1 \]
\[ 2\left(\frac{w}{2}\right) - 1 = 10 \]

**Step 2:**

\[ 55 = (2x - 1)x \]

**Step 3:**

\[ 2x^2 - x = 55 \]
\[ 2x^2 - x - 55 = 0 \]
\[ (2x - 11)(x + 5) = 0 \]
\[ 2x^2 + 10x - 11x - 55 = -x \]
\[ 2x - 11 = 0 \quad x + 5 = 0 \]
\[ +11 \quad -5 \quad -5 \]
\[ x = \frac{11}{2} \quad x = -5 \]

Cannot have a negative width

\[ x = \frac{11}{2} = 5\frac{1}{2} \]

\[ P = 5\frac{1}{2} + 5\frac{1}{2} + 10 + 10 = 31 \]

**Step 4:** The perimeter of the rectangle is 31 meters.
Example #3: A projectile is launched upward from ground level with an initial speed of 98 m/s. How high will it go? When will it return to the ground?

**NOTE:** When a projectile is launched vertically upward with an initial speed \( v \), its height \( h \) above the launch point \( t \) seconds later is given approximately by:

\[
h = vt - 4.9t^2 \quad \text{→ if the distance is measured in meters}
\]

\[
h = vt - 16t^2 \quad \text{→ if the distance is measured in feet}
\]

**Question #2**

\[
0 = 98t - 4.9t^2
\]

\[
t \cdot (98 - 4.9t) = 0
\]

\[
4.9t(20 - t) = 0
\]

\[
\frac{4.9t}{4.9} = 0 \quad 20 - t = 0
\]

\[
t = 0 \text{ sec} \quad t = 20 \text{ sec}
\]

The projectile will hit the ground after 20 sec.

**Question #1**

\[
h = 98(10) - 4.9(10)^2
\]

\[
h = 490 \text{ meters}
\]

The projectile reaches 490 m.